Section 1.8

Sum, Difference, Product and Quotient of Functions

Let f and g be two functions with overlapping domains, then for all x common to both domains, the *sum, difference, product,* and *quotient* of f and g are defined as follows.

a) *Sum:* $(f+g)(x) = f(x) + g(x)$

b) *Difference:* $(f-g)(x) = f(x) - g(x)$

c) *Product:* $(fg)(x) = f(x) \cdot g(x)$

d) *Quotient:* $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

Composition of Two Functions

The composition of the function f with the function g is

$$(f \circ g)(x) = f\big(g(x)\big)$$

The domain of $(f \circ g)$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f.

Problem 1. In the following exercises, find $(f+g)(x), (f-g)(x), (fg)(x), (f/g)(x)$ and $(f/g)(4)$. What is the domain of f/g?

a) $f(x) = 2x - 5,\ g(x) = 2 - x$

b) $f(x) = 2x^2 - 1,\ g(x) = x + 1$

c) $f(x) = \sqrt{x^2 - 4},\ g(x) = \frac{x^2}{x^2+1}$

Problem 2. In the following exercises, find $f \circ g$ and $g \circ f$. Find the domain of each function and each composite function.

a) $f(x) = \sqrt[3]{x-5},\ g(x) = x^3 + 1$

b) $f(x) = |x-4|,\ g(x) = 3 - x$

c) $f(x) = \frac{2}{x^2-4},\ g(x) = x + 4$

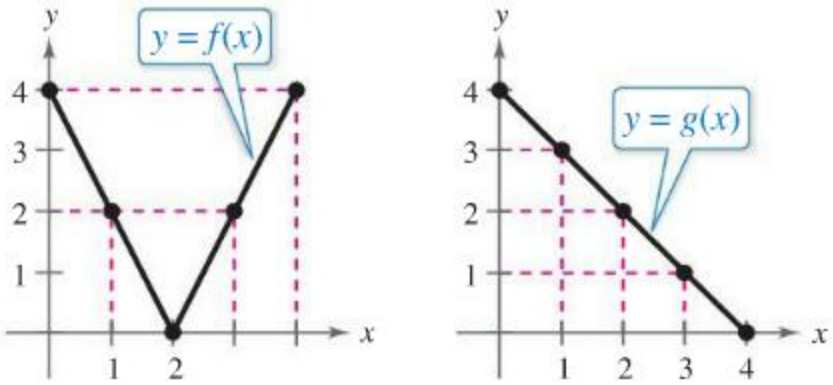
Problem 3. Find two functions f and g such that $(f \circ g)(x) = h(x)$.

a) $h(x) = (4-x)^4$

b) $h(x) = \sqrt{4-x}$

c) $h(x) = \frac{2}{(3x+1)^2}$

Problem 4. Use the graph of f and g to evaluate $(f+g)(3)$, $(f/g)(2)$, $(f \circ g)(1)$, $(f \circ g)(3)$, $(f \circ f)(3)$.

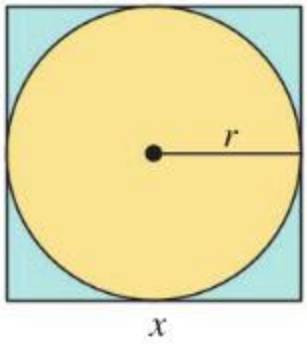


Problem 5. A square concrete foundation is prepared as a base for a cylindrical tank.

a) Write the radius r of the tank as a function of the length x of the sides of the square.

b) Write the area of the circular base of the tank as a function of the radius r.

c) Find and interpret $(A \circ r)(x)$.



Homework: Read section 1.8, do #3, 9, 13, 19, 21, 33, 37, 45, 49 (the quiz for this section will be taken from these problems)